## ECON 496 009 / MATH 493 002 / SYST 465 001 Pricing in Optimization and Game Theory George Mason University Spring 2012

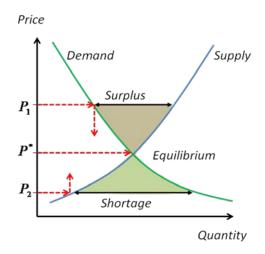
Instructor: Ursula Morris Class Time: Fr. 1:30 - 4:10 Email: **UMorris1@gmu.edu**  Class Room: West 1004 Office Hours: by appointment

#### **Course Description**

Finding the adequate mechanism for pricing limited resources, goods and services is one of the main goals of the theoretical analysis of complex systems. Pricing is one of the driving forces for developing numerical methods to find optimal solutions and economic equilibria. Game theory provides methods to analyze the likely responses of competitors to strategic decisions about prices, expenditures and investments.

The first part of the course will cover the basic ideas and methods in Linear Programming. The fundamental role of pricing in Linear Programming will be emphasized: duality, sensitivity analysis and decomposition are important topics. The introduction of matrix games will show the close relationship between solving the dual pair of an LP and finding an equilibrium in a two person matrix game. The introduction of two-person nonconstant-sum games leads to an understanding of the Nash equilibrium.

After an introduction into nonlinear optimization, the lecture will finish with the demonstration of an algorithm for finding an equilibrium in a linear market using modified barrier functions.



# Prerequisites: OR 441 or Math 441 or a knowledge of the simplex method and the permission of the instructor

### **Course Objectives**

- The students will model linear programming problems and solve them using graphs, the Simplex method, as well as LINDO software.
- The students will perform sensitivity analyses.
- The students will use duality theorems and primal and dual systems to solve linear programming problems.
- The students will formulate and solve two person matrix games using the Simplex and Lemke-Howson algorithms.
- The students will use the Brown-Robinson iterative method for solving matrix games, the Lemke-Howson algorithm to solve bi-matrix games, and the Kornai-Liptak algorithm to solve LP problems with block structure.
- The students will apply the steepest ascent and Newton methods to nonlinear optimization problems.
- The students will solve non-linear optimization problems using the KKT-conditions.
- The students will analyze and apply a mechanism for pricing limited resources.

**Text:** Wayne L. Winston, Operations Research, Applications and Algorithms, Fourth Edition, Thomson, Brooks/Cole 2004.

**Software**: The computational project will use LINDO and LINGO software which is provided with the book by Wayne L. Winston. Free versions of LINDO and LINGO can also be found at *http://www.lindo.com*. Go to this website, click on *Downloads* at the left side and download 'classic LINDO' and LINGO asap.

#### **Course Schedule (tentative):**

	Lecture Topics	Date
1	Introduction and real life applications that led to linear programming; Gauss-Jordan elimination method	1/27/2012
2	Simplex method	
3	Shadow prices and sensitivity analysis.	
4	Duality in LP; basic duality theorems and their economic interpretation; primal-dual systems	
5	Two person matrix games; pure and mixed strategies; John von Neuman's theorem for matrix games	
6	Matrix games and duality in LP; solving matrix games using LP	
7	Midterm	3/9/2012 1:30-4:15
	Springbreak	Mo 3/12/2012–Sun 3/18/2012
8	Lemke-Howson's method for zero-sum matrix	

	games; Prisoner's Dilemma, Nash equilibrium,		
	Lemke-Howson's method for bimatrix games		
9	Brown-Robinson iterative method for solving		
,	matrix games;		
10	LP decomposition according to Kornai and		
10	Liptak		
11	Intro. into nonlinear programming		
12	Method of steepest ascent, Newton's method		
13	Constrained optimization; Lagrange		
15	multipliers; KKT conditions		
14	Equilibrium in a linear exchange market model		
15	Final	5/11/2012	1:30-4:15

A legible paper copy of the homework is due in the beginning of each class. The teacher decides if every homework problem or just a selection of the problems will be graded.

### Grading:

Homework	20%
Midterm	35%
Final Exam	35%
Computational Project	10%

Students are expected to adhere to the George Mason Honor Code.

Make-up midterm exams will be 10% harder.

Class Website: <u>http://classweb.gmu.edu/umorris1</u>